Using the R package collin to visualize the effects of collinearity in distributed lag models

(collin version 0.0.4)

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1 Introduction

This document is a user's guide for the R¹ package collin for the visualization of the effects of collinearity in distributed lag models (DLNM). The package usage is based on two elements provided by the user: a model including a crossbasis created with the dlnm (https://cran.r-project.org/web/packages/dlnm/), and a set of hypothesized true effects. Then, collin performs a simulation study and provides a visualization of results to assess whether the actual results of the study could be driven by collinearity, as described in the original work by Basagaña and Barrera-Gómez [¹]. The illustrative examples used there are reproduced here.

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¹R is a free and open source software and it is available at CRAN (http://cran.r-project.org/).

2 Getting started

The last version released on CRAN can be installed directly within an R session by:

```
install.packages("collin")
```

A brief overview of the package is obtained by:

```
library(collin)

##

## This is collin 0.0.4. For details, use:

## > help(package = 'collin') and browseVignettes('collin')

##

## To cite the methods in the package use:

## > citation('collin')
```

```
help(package = "collin")
```

Once the package has been installed, the vignettes, including the most recent version of this document, as well as the corresponding R code, are available through

```
browseVignettes("collin")
```

3 How does the package work?

The package works in a two-step procedure.

In the first step, the collindlnm function is used to simulate results from a DLNM created with the dlnm package^[2] and a hypothetical effect pattern, both provided by the user. The main arguments of the collindlnm function are:

- model: the fitted DLNM, which includes a crossbasis, to be evaluated. Currently, models allowed are those of class *glm* (i.e. a generalized linear model) or *lme* (i.e. a linear mixed effects model).
- x: a matrix or a vector, depending on whether the hypothetical effect to be explored is linear or non-linear, including the values of the predictor under study.
- cb: an object of class *crossbasis*, included in the model under study (model).
- at: the increase(s) in the predictor under study to be considered to report the effects of the variable. If the hypothetical effect to be analyzed is linear, then it must be a single number. If the hypothetical effect to be analyzed is non-linear, it must be a vector with at least two different values, in order to approximate the shape of the effect.
- cen: the reference value of the predictor of interest, used to calculate effects. If the effect is linear, the value of cen is irrelevant (and it is internally set to 0).

- effect: if the effect is linear, a vector of length (maximum(lag) + 1) including the linear effect at each lag. If the effect is non-linear, a matrix including the effect at each lag (columns) for each value provided in at (rows).
- type: if type = "coef" (default), the hypothetical effect is supposed to be in the linear predictor scale (i.e. it is considered as values of regression coefficient in model). If type = "risk", the effect is supposed to be in terms of relative risks (i.e. exp(coef), as ORs or RRs in logistic or Poisson families, respectively). If model is of class *lme*, then it must be type = "coef" (default).
- shape: the shape of the relationship between the linear predictor and the outcome. Default is, "linear". The case shape = "nonlinear" is currently implemented only if model is of class *qlm*.
- nsim: the number of simulations. Default is 100.
- seed: the seed for reproducibility of results. Default is seed = NULL (no seed).

In the second step, a visualization of the simulation study is displayed using the specific plot() method, which allows to assessing whether the results of the original fitted model are compatible with collinearity problems observed when considering the alternative hypothetical effect pattern. The arguments for plot() depend on the hypothetical effect pattern being linear or non-linear.

For the case of a linear effect, the plot() method requires only two arguments:

- x: a result of the collindlnm.
- lags: indicator of the lags where the results are displayed. Default is lags = NULL, in which case all lags are displayed.

For the case of a non-linear effect, the plot() method requires three additional arguments to allow the user to set how the plots associated at each value of at are shown:

- show: default option, show = "manual", requires the user to manually set the numbers of rows and columns to arrange the plots in a single array of plot, using the par function and setting the value of mfrow. This is the most flexible option to arrange the visualization in a document. The option show = "auto" is the same than show = "manual" except that the value of mfrow is automatically set by the package. The option show = "sequence" shows the plots sequentially, waiting for the user's input before moving to the next plot.
- addlegend and varlegend: to add a label indicating, in each plot, the name of the predictor
 under analysis and the value of at.

4 Illustrative examples

For further details on the following illustrative examples, see the original work ^[1]. Data sets mempm25 and rhospno2, included in the collin package and used in sections 4.1 and 4.2 of this document, are

synthetic data sets generated with the R package synthpop², based on real data sets used in the original work^[1]. Hence, results shown in this document can (and should) differ from the original results.

First, we set the number of simulations and the seed that will be applied to all examples:

```
mynsim <- 100  # number of simulations
myseed <- 23984  # seed</pre>
```

Additional packages required for the examples are:

```
library(nlme) # lme
library(dlnm)

## This is dlnm 2.4.7. For details: help(dlnm) and vignette('dlnmOverview').
library(splines) # ns
```

4.1 Example 1: Windows of susceptibility in a cohort study

Here, we used data from a study by Rivas et al. [3], which aimed to estimate the association between air pollution exposure ($PM_{2.5}$, in $\mu g/m^3$) during the prenatal period and the first seven postnatal years on working memory tests taken at age 8 in a cohort of 2221 children. Exposure matrix contains the exposure to $PM_{2.5}$ at pregnancy, and from years 1 to 7.

```
# data summary:
summary(mempm25)
          id
                   session
                                 school
                                                              agecen
                   1:2221
                            07
                                   : 584
                                            female:4280
                                                                 :-1.87694
##
   0001
               4
                                                          Min.
##
   0002
               4
                   2:2221
                            17
                                    : 496
                                            male :4604
                                                          1st Qu.:-0.75990
               4
                            25
                                                          Median :-0.06722
##
   0003
                   3:2221
                                    : 472
   0004
               4
                   4:2221
                            05
                                    : 440
                                                          Mean : 0.00000
##
                                                          3rd Qu.: 0.71306
   0005
                            32
                                    : 420
##
               4
    0006
               4
                            09
                                    : 412
                                                          Max.
                                                                  : 3.16891
##
##
    (Other):8860
                             (Other):6060
                                                          NA's
                                                                  :13
##
                                             resses
                                                                 pm25y0
                                 :5224
##
   university
                                         Min. :-0.385097
                                                             Min.
                                                                    : 7.169
                                         1st Qu.:-0.159291
                                                             1st Qu.:14.731
##
   secondary
                                 :2628
   primary or less than primary: 988
                                         Median: 0.034258
                                                             Median :16.113
##
   NA's
                                         Mean
                                                : 0.007765
                                                             Mean
                                                                    :16.423
                                         3rd Qu.: 0.163290
##
                                                             3rd Qu.:17.932
##
                                         Max.
                                               : 0.550387
                                                             Max.
                                                                     :30.071
##
                                                             NA's
                                                                     :44
##
        pm25y1
                         pm25y2
                                                            pm25y4
                                           pm25y3
                                                        Min. : 7.574
##
   Min. : 7.406
                     Min. : 7.897
                                       Min. : 7.969
##
   1st Qu.:15.305
                     1st Qu.:15.815
                                       1st Qu.:16.409
                                                        1st Qu.:16.205
   Median :16.564
                     Median :17.277
                                       Median :18.134
                                                        Median :17.818
                                       Mean :18.388
                                                        Mean :18.088
##
   Mean
         :16.879
                           :17.604
                     Mean
```

https://cran.r-project.org/web/packages/synthpop/index.html

```
3rd Qu.:18.271 3rd Qu.:19.252 3rd Qu.:20.114 3rd Qu.:19.849
##
##
   Max.
          :30.235
                    Max. :31.536
                                     Max.
                                             :35.157
                                                       Max.
                                                             :31.832
##
##
        pm25y5
                         pm25y6
                                          pm25y7
                                                            wei
   Min. : 6.272
                    Min. : 5.847
                                    Min. : 5.428
                                                       Min. : 1.051
##
   1st Qu.:14.856
                    1st Qu.:13.483
                                      1st Qu.:12.055
##
                                                       1st Qu.: 1.108
   Median :16.664
                     Median :15.291
                                      Median :13.701
                                                       Median : 1.145
##
##
   Mean
           :16.927
                    Mean
                            :15.402
                                      Mean
                                             :13.967
                                                       Mean
                                                              : 1.329
   3rd Qu.:18.761
##
                     3rd Qu.:17.104
                                      3rd Qu.:15.611
                                                       3rd Qu.: 1.231
           :32.536
                    Max. :27.203
##
   Max.
                                      Max. :32.461
                                                       Max.
                                                             :26.104
##
##
       wmemo
         :-183.43
##
   Min.
##
   1st Qu.: 58.83
   Median: 128.55
##
          : 128.18
   Mean
   3rd Qu.: 189.82
##
   Max.
          : 391.99
   NA's
           :717
# exposure with lags matrix:
pm25lags <- 0:7
nlagspm25 <- length(pm25lags)</pre>
E <- paste0("pm25y", pm25lags)
Qpm25 <- as.matrix(mempm25[, E])</pre>
# exposure pairwise correlations:
corQpm25 <- cor(Qpm25, use = "complete.obs")</pre>
rownames(corQpm25) <- colnames(corQpm25) <- E</pre>
```

)

	Pregnancy	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7
Pregnancy	1.00	0.92	0.89	0.88	0.74	0.61	0.60	0.56
Year 1	0.92	1.00	0.96	0.92	0.79	0.67	0.65	0.62
Year 2	0.89	0.96	1.00	0.92	0.74	0.65	0.59	0.58
Year 3	0.88	0.92	0.92	1.00	0.82	0.61	0.69	0.56
Year 4	0.74	0.79	0.74	0.82	1.00	0.81	0.80	0.82
Year 5	0.61	0.67	0.65	0.61	0.81	1.00	0.76	0.91
Year 6	0.60	0.65	0.59	0.69	0.80	0.76	1.00	0.69
Year 7	0.56	0.62	0.58	0.56	0.82	0.91	0.69	1.00

Table 1: Correlation between $PM_{2.5}$ concentrations at different lags.

The correlation between exposure to $PM_{2.5}$ at different periods, shown in Table 1 is high, with 18% of values exceeding 0.9. Children took the working memory tests in four repeated occasions throughout a year and children were nested in schools, so a 3-level mixed effects model framework was used. We used the distributed lag nonlinear model framework to model the effect of $PM_{2.5}$. We reproduced the original analyses by considering a linear effect of $PM_{2.5}$ and restricting the lagged

effects with a quadratic b-spline with two equally-spaced internal knots. The model was further adjusted for age, sex, maternal education and residential neighborhood socioeconomic status. First, we start with the estimation from single-lag models.

```
# set the exposure increase:
pm25change <- 10
# data.frame to store effects and CI:
pm25effects <- data.frame(lower = rep(NA, nlagspm25),</pre>
                          estimate = rep(NA, nlagspm25),
                           upper = rep(NA, nlagspm25))
# fit models:
for (i in 1:nlagspm25) {
  # select exposure lag:
 Ei <- Qpm25[, i]
  # fit model for that single lag:
  modi <- lme(wmemo ~ Ei + sex + agecen + educ + resses,
              data = mempm25,
              weights = ~ wei,
              random = ~ 1|school/id,
              na.action = na.omit,
              control = lmeControl(opt = "optim"))
  # get effect estimate (for Echange units increase):
 pm25effects[i, ] <- pm25change * intervals(modi)$fixed["Ei", ]</pre>
rm(Ei, modi)
```

A graphical representation of the effects under single-lag models is shown in Figure 1, which has been generated with the following code:

According to Figure 1, models including only $PM_{2.5}$ from a single period showed negative associations between $PM_{2.5}$ and working memory across all periods. Now, we fit the distributed lag model:

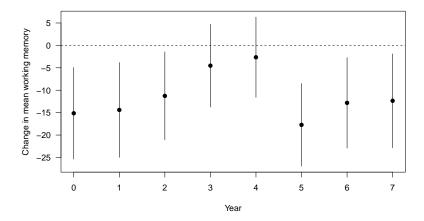


Figure 1: Estimated effect and 95% confidence intervals of a 10 μ g/m³ increase in PM_{2.5} exposure in working memory score across the different time periods, obtained from single-lag models.

A graphical representation of the effects under the previous distributed lag model is shown in Figure 2, which has been generated with the following code:

According to Figure 2, the distributed lag model estimates strong opposing effects. Next, we will check if collinearity is a potential explanation for these results. We first try if the obtained pattern is consistent with a constant effect that has the same cumulative effect than the one obtained. The cumulative effect estimated by the fitted model is stored in the object allfit within the output predmempm25, which was obtained using the crosspred function above. We just need to divide that cumulative effect by the number of lags and use it as the common hypothetical effect at all lags:

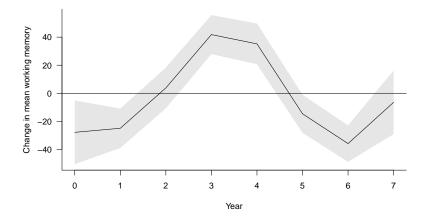


Figure 2: Estimated effect and 95% confidence intervals of a 10 μ g/m³ increase in PM_{2.5} exposure in working memory score across the different time periods, obtained from a distributed lag model.

```
# constant effect (divide cumulative by number of lags):
(conseffpm25 <- rep(predmempm25$allfit / nlagspm25, nlagspm25))

## 10 10 10 10 10 10 10 10

## -3.620734 -3.620734 -3.620734 -3.620734 -3.620734 -3.620734
```

Now we will pass the hypothetical effect to the collindlm function. Since crosspred above was applied to a linear model (specifically, of class lme), the results of the crosspred function are expressed in terms of the regression coefficients of the model. Hence, we need to use collindlm with type = "coef", which is the default option, so we don't need to specify it. Also, we don't need to set the argument shape because in this case the hypothetical effect is linear, which is the default option for shape. Hence, the first step of the procedure is:

```
simconseffpm25 <- collindlnm(model = modmempm25,</pre>
                                        # the original fitted model
                                        # matrix with PM2.5 values at each lag
                      x = Qpm25,
                                        # the crossbasis included in the model
                      cb = cbpm25,
                      at = pm25change,
                                        # increase in PM2.5 to compute effects
                      effect = conseffpm25,
                                        # hypothetical effect
                      nsim = mynsim,
                      seed = myseed)
  ##
  Simulations done.
```

The second step of the procedure uses the plot() method to visualize the results, as shown in Figure 3 using the following call to the plot() method:

```
par(las = 1)
plot(simconseffpm25, xlab = "Year", ylab = "Change in mean working memory")
```

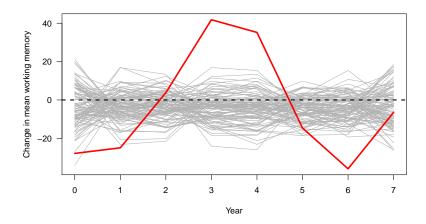


Figure 3: Estimated effect of a $10 \mu g/m^3$ increase in $PM_{2.5}$ exposure across the different time periods over 100 simulations. Estimates from the same simulation run are connected with lines. The red thick line represents the effects observed in the real data set (i.e. original fitted model). Results obtained when simulating a constant effect across all lags, with the cumulative effect being equal to the estimated using the real data.

According to Figure 3, the observed pattern is not consistent with a constant effect at all lags, with the same cumulative effect.

We try now another pattern, in which PM_{2.5} only has a (negative) effect at years 1 and 6, and has no effect at the other years. The effect is 1.5 times the observed cumulative effect:

```
lag1and6effpm25 <- rep(0, nlagspm25)
lag1and6effpm25[c(2, 7)] <- 1.5 * predmempm25$allfit
round(lag1and6effpm25, 2)

## [1] 0.00 -43.45 0.00 0.00 0.00 -43.45 0.00</pre>
```

New simulations under that hypothetical effect:

```
## ........60.......70.......80.......90......100
##
## Simulations done.
```

And the results, shown in Figure 4, are obtained using the plot() method:

```
par(las = 1)
plot(simlag1and6effpm25, xlab = "Year", ylab = "Change in mean working memory")
```

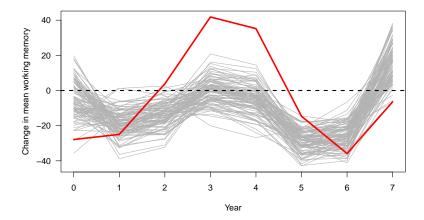


Figure 4: Estimated effect of a $10~\mu\mathrm{g/m^3}$ increase in PM_{2.5} exposure across the different time periods over 100 simulations. Estimates from the same simulation run are connected with lines. The red thick line represents the effects observed in the real data set (i.e. original fitted model). Results obtained when simulating a real effect of years 1 and 6 (1.5 times the size of the cumulative effect estimated by the original model) and no effect of all other periods.

The resulting curves in Figure 4 are not consistent with the observed pattern either. Finally, we try another pattern, one in which $PM_{2.5}$ only has a (negative) effect at lag 5, and has no effect on the other lags. The effect is four times the observed cumulative effect:

```
lag5seffpm25 <- rep(0, nlagspm25)
lag5seffpm25[6] <- 4 * predmempm25$allfit
round(lag5seffpm25, 2)
## [1] 0.00 0.00 0.00 0.00 -115.86 0.00 0.00</pre>
```

New simulations under that hypothetical effect:

And the results, shown in Figure 5, are obtained using the plot() method:

```
par(las = 1)
plot(simlag5effpm25, xlab = "Year", ylab = "Change in mean working memory")
```

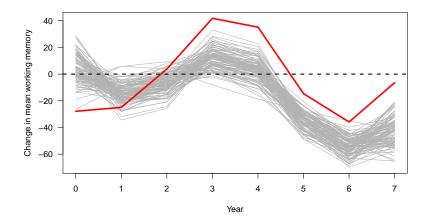


Figure 5: Estimated effect of a $10 \ \mu g/m^3$ increase in PM_{2.5} exposure across the different time periods over 100 simulations. Estimates from the same simulation run are connected with lines. The red thick line represents the effects observed in the real data set (i.e. original fitted model). Results obtained when simulating a real effect of year 5 (four times the size of the cumulative effect estimated by the original model) and no effect of all other periods.

Based on the observed results, this is a non-intuitive scenario. It was obtained after exploring all possibilities in which there is an effect only in one of the periods. The scenario with an effect only at year 5 (Figure 5) reproduced the most similar pattern to the observed one. This is an interesting scenario because fitting the specified distributed lag model generates positive estimates at years 3 and 4. The magnitude of the simulated effect (four times the observed cumulative effect) was important, as only with large effects were we able to observe large deviations in the opposite direction. However, even this scenario that generated similar curves to the observed pattern did not generate curves as extreme as the observed one. Thus, we should conclude that this particular alternative scenario is not compatible with the data.

Still, the example in Figure 5 shows a scenario in which a poor choice of function to constrain lagged associations (smoothed lagged effects are not a good choice if exposure is associated with the

outcome at only one period) in combination with the correlation between exposure at different times (collinearity) and strong signal to noise ratio will lead to estimates that would report non-existent negative effects at some years. Given the direction and magnitude of those biases, one could not discard the possibility that a bad choice of constraining functions combined with collinearity may have a role in explaining the unexpected positive results.

In reality, however, no one knows the true data generating mechanism, which makes the choice of the appropriate lag function difficult.

4.2 Example 2: Time series study with linear effects

In this example, we analyzed the relationship between the daily number of hospital admissions for respiratory causes and ambient NO₂ concentrations (in $\mu g/m^3$) in the city of Barcelona (Spain) for years 2006-2015:

```
summary(rhospno2)
##
         date
                                                                 dow
                                               year
##
   Min.
           :2006-01-01
                          Min.
                                 : 366
                                          Min.
                                                 :2006
                                                          Sunday
                                                                   :522
   1st Qu.:2008-07-01
                          1st Qu.:1279
                                          1st Qu.:2008
                                                         Monday
                                                                   :522
##
   Median :2010-12-31
                          Median:2192
                                          Median:2010
                                                          Tuesday
                                                                  :522
##
   Mean
           :2010-12-31
                          Mean
                                 :2192
                                          Mean
                                                 :2010
                                                          Wednesday: 522
##
   3rd Qu.:2013-07-01
                          3rd Qu.:3104
                                          3rd Qu.:2013
                                                         Thursday:522
           :2015-12-31
                                 :4017
                                                 :2015
                                                         Friday
##
   Max.
                          Max.
                                          Max.
                                                                   :521
##
                                                          Saturday:521
##
         temp
                          no2
                                           hresp
##
   Min.
           : 1.40
                            : 3.00
                                             : 8.00
                    Min.
                                      Min.
##
   1st Qu.:11.78
                    1st Qu.: 46.00
                                      1st Qu.: 26.00
   Median :16.80
                    Median : 60.00
                                      Median: 34.00
##
                    Mean
##
   Mean
           :16.96
                            : 61.34
                                      Mean
                                             : 35.88
##
   3rd Qu.:22.40
                    3rd Qu.: 74.00
                                      3rd Qu.: 43.00
##
   Max.
           :30.40
                    Max.
                            :159.00
                                              :103.00
                                      Max.
##
                    NA's
                            :77
```

We used the DLNM framework with a generalized linear model with the quasi-Poisson family to allow for overdispersion. In particular, we assumed the effect of NO₂ to be linear (in the log scale), explored lagged effects of up to 14 days, and constrained the lag function to follow a natural spline with three internal knots equally-spaced in the log scale. The model was further adjusted for day of the week, temperature (using a crossbasis with a natural spline with 4 equally-spaced internal knots to model the non-linear effects of temperature, and a natural spline with 3 internal knots equally-spaced on the log scale to model the lag structure up to lag 21), and for trend and seasonality (using a natural spline of time with 7 degrees of freedom per year).

First, we need to create the matrix of the lagged values of the exposure, which can be done using the lagpad function. This function has two arguments: x, the numeric vector to be lagged, and k, the number of lags to be applied:

```
# create matrix with lagged data:
nlagsno2 <- 15 # number of lags considered (14 + 1)</pre>
```

```
Qno2 <- matrix(NA, nrow = dim(rhospno2)[1], ncol = nlagsno2)
for (i in 1:nlagsno2)
   Qno2[, i] <- lagpad(x = rhospno2$no2, k = i - 1)

# correlation betweeen exposures:
corQno2 <- cor(Qno2, use = "complete.obs")
rownames(corQno2) <- colnames(corQno2) <- paste0("lag", 0:(nlagsno2 - 1))</pre>
```

```
print(corQno2, digits = 2)
```

	Given day	-1 d.	-2 d.	-3 d.	-4 d.	-5 d.	-6 d.	-7 d.	-8 d.	-9 d.	-10 d.	-11 d.	-12 d.	-13 d.	-14 d.
Given day	1.00	0.62	0.33	0.22	0.18	0.21	0.30	0.38	0.27	0.13	0.08	0.09	0.12	0.23	0.32
-1 d.	0.62	1.00	0.62	0.33	0.22	0.19	0.21	0.31	0.38	0.27	0.14	0.09	0.09	0.13	0.24
-2 d.	0.33	0.62	1.00	0.62	0.34	0.23	0.19	0.21	0.31	0.39	0.28	0.14	0.09	0.09	0.13
-3 d.	0.22	0.33	0.62	1.00	0.62	0.34	0.23	0.20	0.22	0.31	0.39	0.28	0.15	0.10	0.10
-4 d.	0.18	0.22	0.34	0.62	1.00	0.62	0.34	0.23	0.20	0.22	0.31	0.40	0.29	0.15	0.11
-5 d.	0.21	0.19	0.23	0.34	0.62	1.00	0.62	0.35	0.23	0.20	0.22	0.32	0.40	0.29	0.16
-6 d.	0.30	0.21	0.19	0.23	0.34	0.62	1.00	0.62	0.35	0.23	0.20	0.22	0.32	0.40	0.30
-7 d.	0.38	0.31	0.21	0.20	0.23	0.35	0.62	1.00	0.62	0.35	0.24	0.20	0.23	0.33	0.41
-8 d.	0.27	0.38	0.31	0.22	0.20	0.23	0.35	0.62	1.00	0.62	0.34	0.23	0.20	0.23	0.33
-9 d.	0.13	0.27	0.39	0.31	0.22	0.20	0.23	0.35	0.62	1.00	0.62	0.34	0.23	0.20	0.23
-10 d.	0.08	0.14	0.28	0.39	0.31	0.22	0.20	0.24	0.34	0.62	1.00	0.62	0.35	0.23	0.20
-11 d.	0.09	0.09	0.14	0.28	0.40	0.32	0.22	0.20	0.23	0.34	0.62	1.00	0.63	0.35	0.24
-12 d.	0.12	0.09	0.09	0.15	0.29	0.40	0.32	0.23	0.20	0.23	0.35	0.63	1.00	0.63	0.35
-13 d.	0.23	0.13	0.09	0.10	0.15	0.29	0.40	0.33	0.23	0.20	0.23	0.35	0.63	1.00	0.63
-14 d.	0.32	0.24	0.13	0.10	0.11	0.16	0.30	0.41	0.33	0.23	0.20	0.24	0.35	0.63	1.00

Table 2: Correlation between NO₂ concentrations at different lags.

The correlation between NO_2 concentrations at different lags, shown in Table 2, were lower than in the previous example (Table 1), with highest values around 0.6 for adjacent days.

Now, we start the modelling with the estimates when including single lags in the model:

```
lower = rep(NA, nlagsno2),
                               upper = rep(NA, nlagsno2))
for (i in 1:nlagsno2) {
  # select exposure lag:
  Ei <- Qno2[, i]</pre>
  # fit model:
  modi <- glm(hresp ~ Ei + cbtemp + ns(t, 7 * nyears) + dow,
               data = rhospno2,
               family = quasipoisson,
               na.action = na.exclude)
  # get beta estimates and CI:
  ints <- confint.default(modi)</pre>
  coefsno2single$lower[i] <- ints["Ei", "2.5 %"]</pre>
  coefsno2single$estimate[i] <- summary(modi)$coefficients["Ei", "Estimate"]</pre>
  coefsno2single$upper[i] <- ints["Ei", "97.5 %"]</pre>
# set the exposure increase:
no2change <- 10
# compute effects (RRs):
effectno2single <- exp(no2change * coefsno2single)
```

A graphical representation of the effects under single-lag models is shown in Figure 6, which has been generated with the following code:

According to Figure 6, single-lag models showed significant increases in risk of respiratory hospital admission (i.e. relative risk, RR > 1) at lags 0 and 6, other periods with elevated non-significant RRs, and a non-significant RR < 1 at lags 1 and 2. Now, we fit the distributed lag model to the data:

In this case in which we are going to include two crossbases in the model that will be passed to collindlnm, it gives problems because of the names:

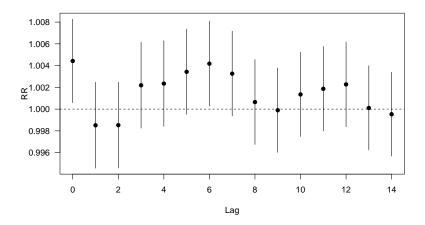


Figure 6: Estimated relative risk (RR) and 95% confidence intervals of hospital admission for respiratory causes for a 10 $\mu \rm g/m^3$ increase in ambient NO₂ concentration across the different time periods, obtained from single-lag models.

```
colnames(cbtemp)

## [1] "v1.11" "v1.12" "v1.13" "v1.14" "v1.15" "v2.11" "v2.12" "v2.13" "v2.14"

## [10] "v2.15" "v3.11" "v3.12" "v3.13" "v3.14" "v3.15" "v4.11" "v4.12" "v4.13"

## [19] "v4.14" "v4.15" "v5.11" "v5.12" "v5.13" "v5.14" "v5.15"

colnames(cbno2)

## [1] "v1.11" "v1.12" "v1.13" "v1.14" "v1.15"

all(colnames(cbno2) %in% colnames(cbtemp))

## [1] TRUE
```

To solve it, we need to change the names of one of the crossbasis:

```
# change the names of the crossbassis for temperature:
aux <- as.data.frame(cbtemp)</pre>
ncbtemp <- dim(cbtemp)[2]</pre>
crosstempnames <- pasteO("crosstemp", 1:ncbtemp)</pre>
names(aux) <- crosstempnames</pre>
rhospno2 <- cbind(rhospno2, aux)</pre>
rm(aux)
names(rhospno2)
##
    [1] "date"
                                        "year"
                                                       "dow"
                                                                       "temp"
    [6] "no2"
                                        "crosstemp1"
                                                       "crosstemp2"
                        "hresp"
                                                                       "crosstemp3"
## [11] "crosstemp4" "crosstemp5"
                                        "crosstemp6"
                                                                      "crosstemp8"
                                                       "crosstemp7"
```

```
## [16] "crosstemp9" "crosstemp10" "crosstemp11" "crosstemp12" "crosstemp13"
## [21] "crosstemp14" "crosstemp15" "crosstemp16" "crosstemp17" "crosstemp18"
## [26] "crosstemp19" "crosstemp20" "crosstemp21" "crosstemp22" "crosstemp23"
## [31] "crosstemp24" "crosstemp25"
```

Now we can fit the model with the two crossbases:

```
# model formula:
formhosp <- paste0("hresp ~ cbno2 + ",</pre>
                   paste(crosstempnames, collapse = " + "),
                    " + ns(t, 7 * nyears) + dow")
(formhosp <- as.formula(formhosp))</pre>
## hresp ~ cbno2 + crosstemp1 + crosstemp2 + crosstemp3 + crosstemp4 +
       crosstemp5 + crosstemp6 + crosstemp7 + crosstemp8 + crosstemp9 +
##
       crosstemp10 + crosstemp11 + crosstemp12 + crosstemp13 + crosstemp14 +
##
##
       crosstemp15 + crosstemp16 + crosstemp17 + crosstemp18 + crosstemp19 +
##
       crosstemp20 + crosstemp21 + crosstemp22 + crosstemp23 + crosstemp24 +
       crosstemp25 + ns(t, 7 * nyears) + dow
##
# fit model:
modrhospno2 <- glm(formhosp, family = quasipoisson, na.action = na.exclude, data = rhospno2)
# predict effects at different lags:
predrhospno2 <- crosspred(basis = cbno2, model = modrhospno2, cen = 0, at = no2change)</pre>
```

A graphical representation of the effects under the previous distributed lag model is shown in Figure 7, which has been generated with the following code:

According to Figure 7, when fitting the distributed lag model, there was a statistically significant increase in respiratory hospital admissions associated with levels of NO₂ at lag 0, followed by a statistically significant decreased risk at lags 1 and 2, and a subsequent statistically significant increase around lag 5. The decrease in risk at lags 1 and 2 could be consistent with the harvesting or short-term mortality displacement phenomenon (details in the original work ^[1]). However, there is also the possibility that this decrease in risk and the subsequent increases around lag 5 could be explained by collinearity since, as we showed above, collinearity can induce estimates with opposing signs. To explore its plausibility, we will analyze a hypothetical truth in which the real effect exists only at lag 0, with the same size as the estimated by the fitted model.

```
# Effect (RRs) only at lags 0, same as observed

RRveclag0 <- rep(1, nlagsno2)

RRveclag0[1] <- predrhospno2$matRRfit[, "lag0"]

RRveclag0

## [1] 1.006564 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 1.000000 ## [9] 1.000000 1.000000 1.000000 1.000000 1.000000
```

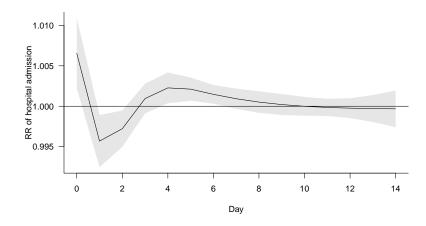


Figure 7: Estimated relative risk (RR) and 95% confidence intervals of hospital admission for respiratory causes for a 10 μ g/m³ increase in ambient NO₂ concentration across the different time periods, obtained from a distributed lag model.

Now we pass the hypothetical effect to collindlnm. Since it is given as RRs, we need to set type = "risk":

The results, shown in Figure 8, are obtained using the plot() method:

```
par(las = 1)
plot(simlagOeffno2, xlab = "Day", ylab = "RR of hospital admission")
```

Results displayed in Figure 8 show that, under a hypothetical truth in which only lag 0 has a real effect, the pattern of the estimated effects bear some similarity to those obtained with the real data (red line), so that collinearity could be involved in these results. I.e. even under the situation in which only lag 0 has a real effect, distributed lag models can suggest a reduction in risk at lags 1-2 and subsequent increases in risk around lag 5. It is important to note that the observed pattern is compatible with many real scenarios, and in particular it is also compatible with a scenario with

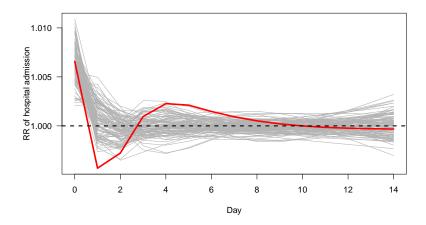


Figure 8: Estimated relative risk (RR) of hospital admission for respiratory cause for a $10~\mu g/m^3$ increase in ambient NO₂ concentration across different lags, obtained from a distributed lag model, over 100 simulations. Estimates from the same simulation run are connected with lines. The results were obtained when simulating an effect only at lag 0 and of the same magnitude as the estimated with the real data. The red thick line represents the RRs estimated with the real data set.

a real increase in risk at lag 0 and a real decrease in risk at lag 2 (e.g. because of the harvesting phenomenon) (details in the original work^[1]).

4.3 Example 3: Time series study with nonlinear effects

In this example, we analyzed the relationship between daily mortality and ambient temperature in Chicago from 1987 to 2000. These data are available as part of the R package dlnm:

```
chica <- chicagoNMMAPS[, c("date", "time", "year", "dow", "death", "temp", "pm10")]</pre>
summary(chica)
##
         date
                                time
                                                year
                                                                  dow
##
           :1987-01-01
                          Min.
                                  :
                                          Min.
                                                  :1987
                                                          Sunday
                                                                    :731
                                      1
    1st Qu.:1990-07-02
                          1st Qu.:1279
                                          1st Qu.:1990
                                                          Monday
##
                                                                    :730
##
   Median :1993-12-31
                          Median:2558
                                          Median:1994
                                                          Tuesday
                                                                   :730
##
           :1993-12-31
                          Mean
                                  :2558
                                          Mean
                                                  :1994
                                                          Wednesday:730
    3rd Qu.:1997-07-01
                          3rd Qu.:3836
                                          3rd Qu.:1997
                                                          Thursday:731
##
##
    Max.
           :2000-12-31
                          Max.
                                  :5114
                                          Max.
                                                  :2000
                                                          Friday
                                                                    :731
##
                                                          Saturday:731
##
        death
                          temp
                                             pm10
##
   Min.
           : 69.0
                     Min.
                             :-26.667
                                               : -3.05
                                        Min.
##
    1st Qu.:105.0
                     1st Qu.:
                              1.667
                                        1st Qu.: 20.77
##
   Median :114.0
                     Median : 10.556
                                        Median : 30.25
          :115.4
                     Mean
                            : 10.107
                                        Mean : 33.74
   Mean
                     3rd Qu.: 19.444
    3rd Qu.:124.0
                                        3rd Qu.: 42.42
##
```

```
## Max. :411.0 Max. : 33.333 Max. :356.18
## NA's :251
```

First, we calculate the matrix of lagged values of temperature:

```
# create matrix with lagged data:
nlagstemp <- 31  # number of lags considered (30 + 1)

Qtemp <- matrix(NA, nrow = dim(chica)[1], ncol = nlagstemp)
for (i in 1:nlagstemp) {
   Qtemp[, i] <- lagpad(x = chica$temp, k = i - 1)
}
colnames(Qtemp) <- pasteO("lag", 0:(nlagstemp - 1))

# correlation betweeen exposures
corQtemp <- cor(Qtemp, use = "complete.obs")
rownames(corQtemp) <- colnames(corQtemp) <- pasteO("lag", 0:(nlagstemp - 1))</pre>
```

The correlation between temperature in two consecutive days is 0.94, the correlation is still greater than 0.8 for days separated by 8 days or less, and it is around 0.7 for a 30-day separation. We used the distributed lag nonlinear model framework, with the same specifications used in the vignette of the dlnm package, to model the association between mortality and temperature. [2] Namely, we used a crossbasis for temperature, using a quadratic b-spline with 3 equally-spaced internal knots to model the exposure-response association, and a natural spline with 3 equally-spaced internal knots in the log space to model the lagged association up to lag 30. The quasi-Poisson regression model included as additional covariates day of the week, PM₁₀ concentrations (modeled with a crossbasis assuming linear effects and a strata lag structure up to lag 1), and a control for trends and seasonality with a natural spline of time with 7 degrees of freedom per year.

First, we create the crossbasis for PM_{10} :

Now, we start the modelling with the estimates when including single lags in the model:

```
# reference value of temperature for effects calculation:
centemp <- 21

# evaluation points (values of temperature):
attemp <- c(-20, 0, 33)</pre>
```

```
# get beta coefficients and CI for each model:
coefs <- lower <- upper <- matrix(NA, nrow = dim(Qtemp)[2], ncol = length(attemp))</pre>
# number of years for time spline:
nyearschica <- diff(range(chica$year, na.rm = TRUE)) + 1</pre>
for (i in 1:nlagstemp) {
  Ei <- Qtemp[, i]</pre>
  # crossbasis for lag i of temperature:
  cbi <- onebasis(Ei, fun = "bs", knots = ktemp, degree = 2)
  modi <- glm(death ~ cbi + baspm + ns(time, 7 * nyearschica) + dow,
               data = chica,
               family = quasipoisson)
  # get effect estimates and CI:
  predi <- crosspred(basis = cbi, model = modi, at = attemp, cen = centemp)</pre>
  lower[i, ] <- t(predi$matRRlow)</pre>
  coefs[i, ] <- t(predi$matRRfit)</pre>
  upper[i, ] <- t(predi$matRRhigh)</pre>
```

A graphical representation of the effects under single-lag models is shown in Figure 9, which has been generated with the following code:

Figure 9 shows that mortality risk increased with cold temperatures for lags < 10 days, except for lag 0, which even showed a protective effect at 0°C (compared to 21°C). For heat, increased risks during the first four days were observed, followed by some lags with protective effects. Now fit the distributed lag model to the data:

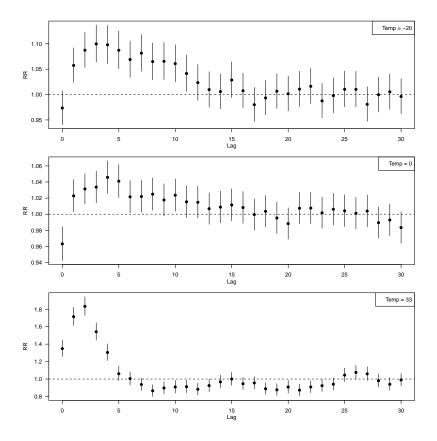


Figure 9: Relative risks (RR) and 95% confidence intervals for the associations between temperature and mortality by lag, using single-lag models. Results are presented for temperatures -20°C, 0°C, 33°C, taking 21°C as a reference. The effect of temperature was modeled using a quadratic b-spline with 3 equally-spaced internal knots.

A graphical representation of the effects under the previous distributed lag model is shown in Figure 10, which has been generated with the following code:

```
par(las = 1, mfrow = c(3, 1), mar = c(4, 4, 0, 2) + 0.1)
plot(predtemp, var = attemp[1])
legend("topright", paste0("Temp = ", attemp[1]))

plot(predtemp, var = attemp[2], yaxt = "n", ylim = c(0.94, 1.05))
axis(2, at = c(0.96, 0.98, 1, 1.02, 1.04))
legend("topright", paste0("Temp = ", attemp[2]))
```

```
plot(predtemp, var = attemp[3])
legend("topright", paste0("Temp = ", attemp[3]))
```

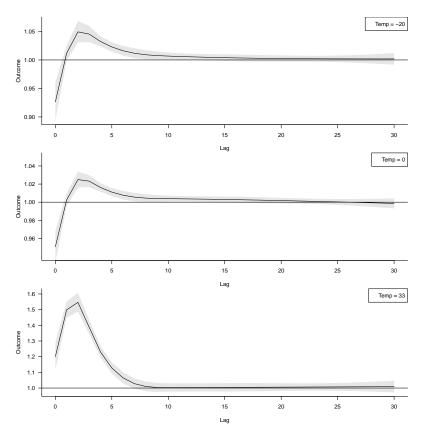


Figure 10: Relative risks (RR) and 95% confidence intervals for the associations between temperature and mortality by lag, using a distributed lag model. Results are presented for temperatures -20°C, 0°C, 33°C, taking 21°C as a reference. The effect of temperature was modeled using a crossbasis with a quadratic b-spline with 3 equally-spaced internal knots for the exposure-response association and a natural spline with 3 equally-spaced internal knots in the log space to model the lagged association.

According to Figure 10, associations were similar but more precise than those from single-lag models (Figure 9). A protective association at lag 0 was detected at both -20°C and 0°C. Now, we analyze the scenario in which there were no true RRs below one:

```
## lag14 lag15 lag16 lag17 lag18 lag19 lag20 lag21 lag22 lag23 lag24 lag25
               1
                    1
                         1 1.00 1.00 1.00 1.00 1.00 1.00 1.00
## 0
         1
               1
                     1
                           1 1.00 1.00 1.00 1.00 1.00 1.00 1.00
## 33
          1
               1
                     1
                           1 1.01 1.01 1.01 1.01 1.01 1.01 1.01
      lag26 lag27 lag28 lag29 lag30
## -20 1.00 1.00 1.00 1.00 1.00
## 0
       1.00 1.00 1.00 1.00 1.00
## 33
      1.01 1.01 1.01 1.01 1.01
attempc <- as.character(attemp)</pre>
attempc
                 "33"
## [1] "-20" "0"
# all effects null from lag 8 included
RRmattemp[, paste0("lag", 6:(nlagstemp - 1))] <- 1</pre>
# at temp 1:
RRmattemp[attempc[1], paste0("lag", 0:2)] <- 1</pre>
RRmattemp[attempc[1], paste0("lag", 3:5)] <- c(1.07, 1.12, 1.06)
# at temp 2:
RRmattemp[attempc[2], paste0("lag", 0:2)] <- 1</pre>
RRmattemp[attempc[2], paste0("lag", 3:5)] <- c(1.08, 1.03, 1.01)
# at temp 3:
RRmattemp[attempc[3], paste0("lag", 0:5)] <- c(1.15, 1.20, 1.22, 1.15, 1.10, 1.04)
RRmattemp
##
      lag0 lag1 lag2 lag3 lag4 lag5 lag6 lag7 lag8 lag9 lag10 lag11 lag12 lag13
## -20 1.00 1.0 1.00 1.07 1.12 1.06 1 1 1
                                                       1
                                                            1
## 0 1.00 1.0 1.00 1.08 1.03 1.01
                                    1 1
                                              1
                                                   1
                                                        1
                                                              1
                                   1
## 33 1.15 1.2 1.22 1.15 1.10 1.04
                                        1
                                              1
                                                   1
                                                        1
      lag14 lag15 lag16 lag17 lag18 lag19 lag20 lag21 lag22 lag23 lag24 lag25
## -20
        1
              1
                    1
                          1
                               1
                                    1
                                           1
                                                1
                                                      1
## 0
                     1
                           1
                                1
                                      1
                                           1
                                                 1
                                                       1
                                                            1
                                                                  1
                                                                        1
          1
               1
          1
               1
                     1
                           1
                                1
                                      1
                                           1
                                                 1
                                                       1
                                                                        1
     lag26 lag27 lag28 lag29 lag30
## -20
                     1
         1
              1
                           1
## 0
          1
               1
                     1
                           1
                                1
## 33
                  1
             1
                        1
```

Now we need to set type = "risk" (because we have RRs) and shape = "nonlinear":

The results, shown in Figure 11, are obtained using the plot() method:

```
par(las = 1, mfrow = c(3, 1), mar = c(4, 4, 2, 2))
plot(simchicalagOnull, varlegend = "Temperature")
```

The plot() method also allows the user to select a subset of lags to be shown, using the argument lags (by default, all lags are shown). Also, we can set show = "auto" to let the grid plot be arranged automatically. For instance, the following code produces Figure 12:

```
par(las = 1)
plot(simchicalag0null, lags = 0:8, show = "auto", varlegend = "Temperature")
```

The gray lines in Figure 11 show the results obtained when data were simulated from a scenario in which there were no true RRs below one. Hence, results obtained in that scenario could be compatible with the estimated effects using the real data, i.e. RR < 1 at lag 0 for cold temperatures and RR < 1 at the second week for hot temperatures. Still, even after exploring several potential scenarios, the observed results lay at the extreme of the obtained distribution. This, and the fact that single-lag models also show RR < 1 at lag 0 for cold temperatures, suggest that there might be other explanations for this result.

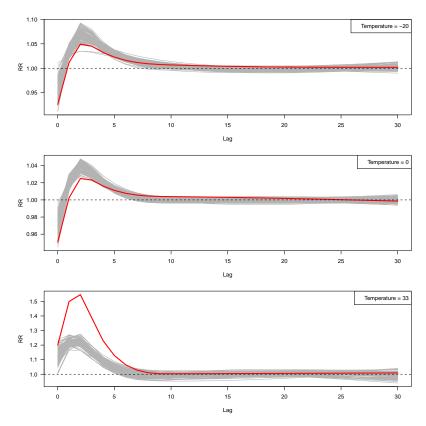


Figure 11: Estimated relative risks (RR) for mortality as a function of temperature obtained from distributed lag models, over 100 simulations. Estimates from the same simulation run are connected with gray lines. The red thick line represents the RRs observed in the real data set. Results are presented for temperatures -20°C, 0°C, 33°C, taking 21°C as a reference. The results were obtained when simulating data with the following RRs: At temperature -20°C: RR = 1 at lags 0-2, and 6-30, RR = 1.07 at lag 3, RR = 1.12 at lag 4 and RR = 1.06 at lag 5; at temperature 0°C: RR = 1 for lags 0-2 and 6-30, RR = 1.08 at lag 3, RR = 1.03 at lag 4, RR 1.01 at lag 5; at temperature 33°C: RR = 1.15 at lag 0, RR = 1.2 at lag 1, RR 1.22 at lag 2, RR = 1.15 at lag 3, RR = 1.10 at lag 4, RR = 1.04 at lag 5 and RR = 1 at lags 6-30.

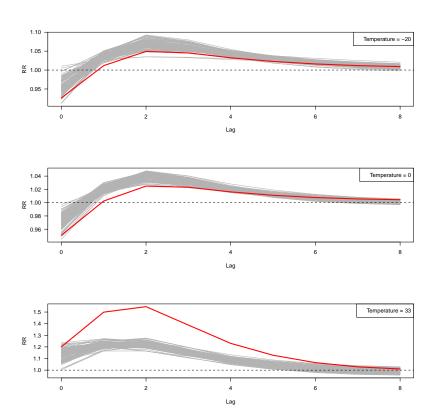


Figure 12: (Same than Figure 11 but showing until lag 10). Estimated relative risks (RR) for mortality as a function of temperature obtained from distributed lag models, over 100 simulations. Estimates from the same simulation run are connected with gray lines. The red thick line represents the RRs observed in the real data set. Results are presented for temperatures -20°C, 0°C, 33°C, taking 21°C as a reference. The results were obtained when simulating data with the following RRs: At temperature -20°C: RR = 1 at lags 0-2, and 6-30, RR = 1.07 at lag 3, RR = 1.12 at lag 4 and RR = 1.06 at lag 5; at temperature 0°C: RR = 1 for lags 0-2 and 6-30, RR = 1.08 at lag 3, RR = 1.03 at lag 4, RR 1.01 at lag 5; at temperature 33°C: RR = 1.15 at lag 0, RR = 1.2 at lag 1, RR 1.22 at lag 2, RR = 1.15 at lag 3, RR = 1.10 at lag 4, RR = 1.04 at lag 5 and RR = 1 at lags 6-30.

Bibliography

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- [3] I. Rivas, X. Basagaña, M. Cirach, M. López-Vicente, E. Suades-González, R. Garcia-Esteban, M. Álvarez-Pedrerol, P. Dadvand, and J. Sunyer. Association between early life exposure to air pollution and working memory and attention. *Environmental Health Perspectives*, 127(5):57002, 2019. URL https://doi.org/10.1289/EHP3169.